Bounded Indistinguishability for Simple Sources

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⇧
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$X|_{a_1+\cdots+a_n=0}$ and $X|_{a_1+\cdots+a_n=1}$ are $(n-1)$-wise indistinguishable

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k-wise independence: **derandomization**

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AC^0 reconstruction requires k-wise indistinguishability

secure multiparty computation and **leakage-resilience** require share manipulation
breaks k-wise independence but not k-wise indistinguishability

Braverman for indistinguishability?

[Bogdanov–Ishai–Viola–Williamson 2016]

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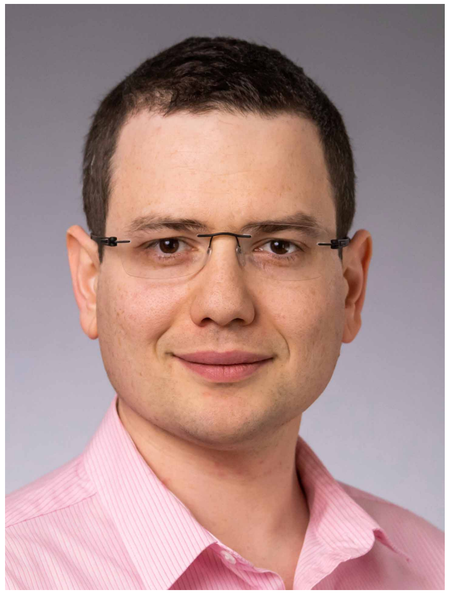

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polylog independence
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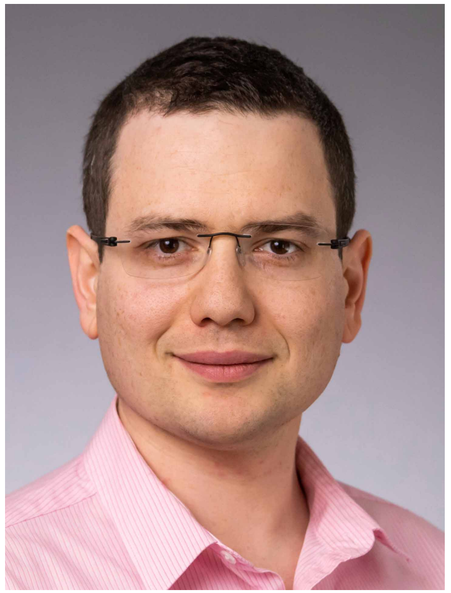

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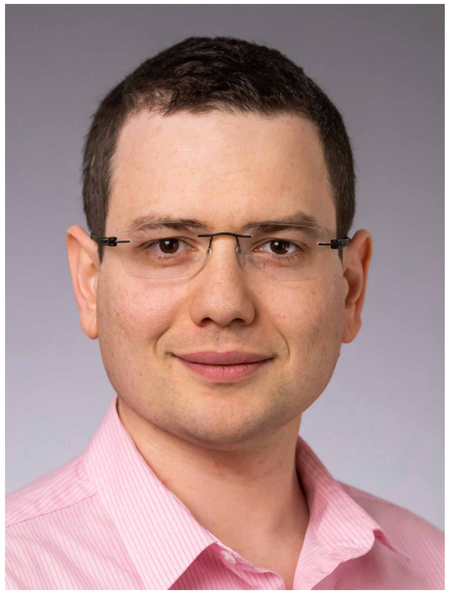

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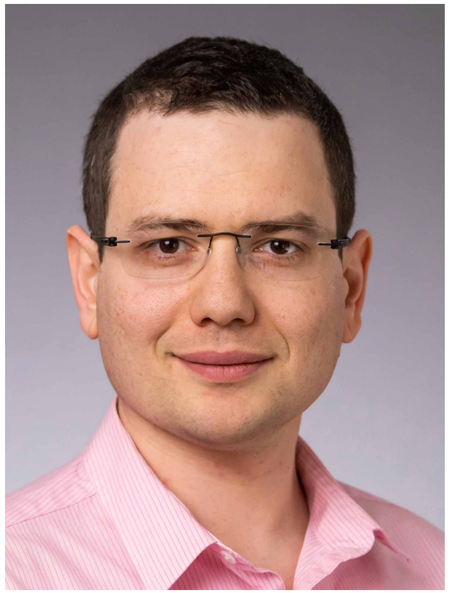

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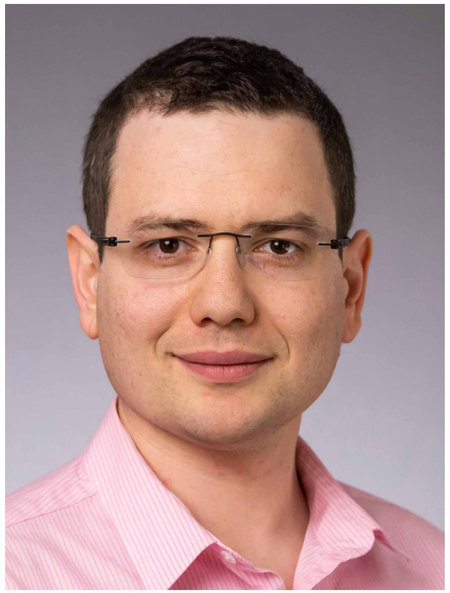

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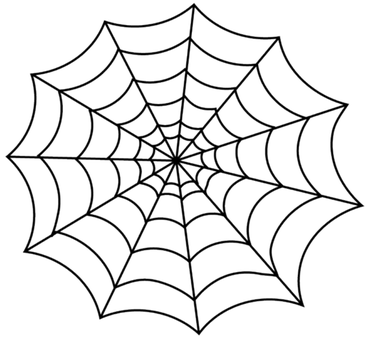
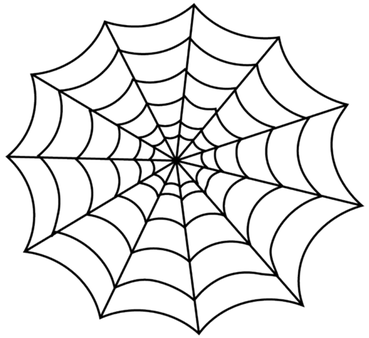
Some instances reducible to Braverman; others (e.g. LDPC codes) not



Web of Conjectures

Given: class of sources (e.g. affine), class of circuits (e.g. AC^0)



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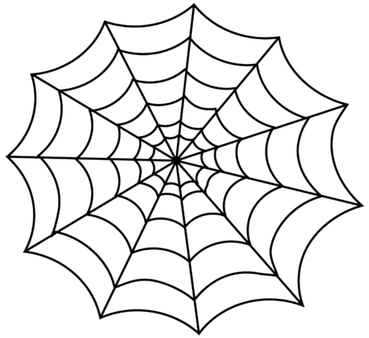

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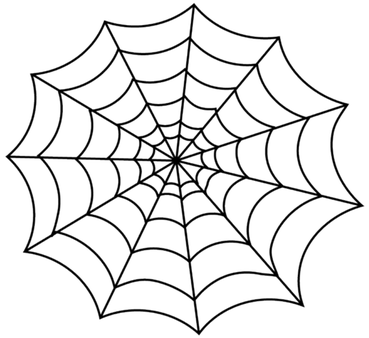

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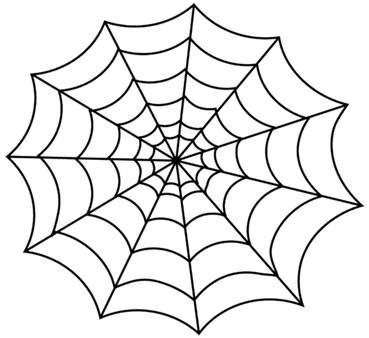

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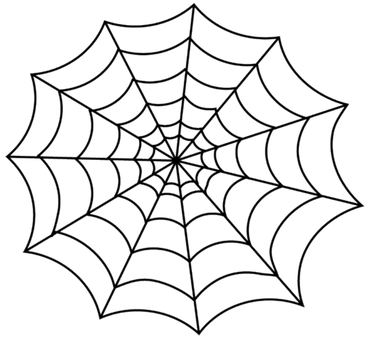

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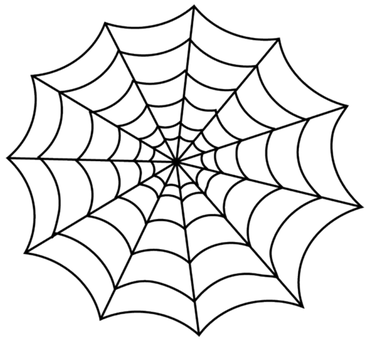
Special case: compute parity of codewords belonging to LDPC code



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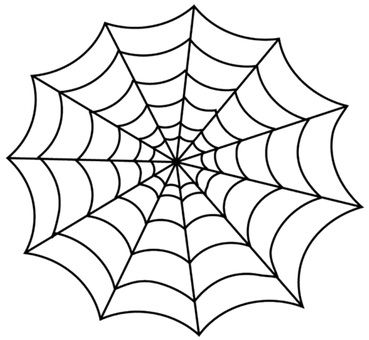

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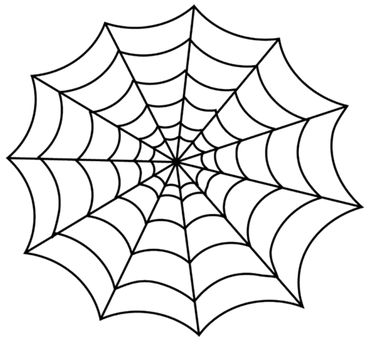

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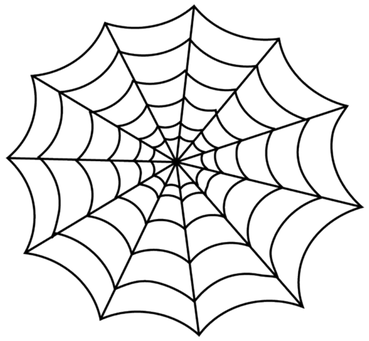

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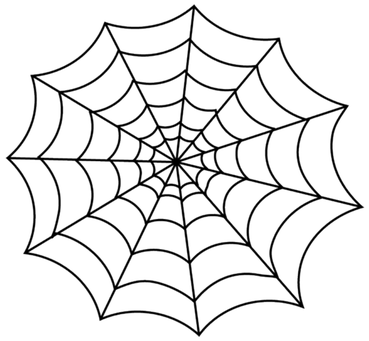

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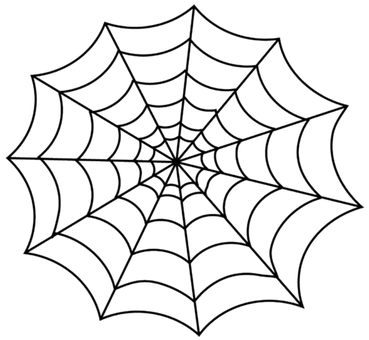

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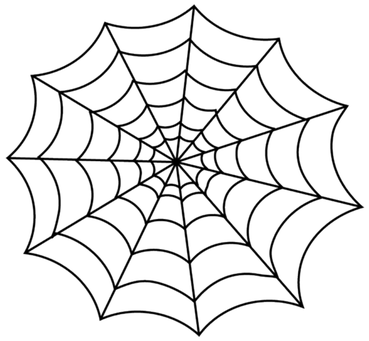

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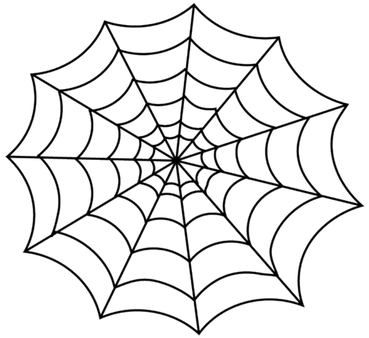
Arbitrary preprocessing



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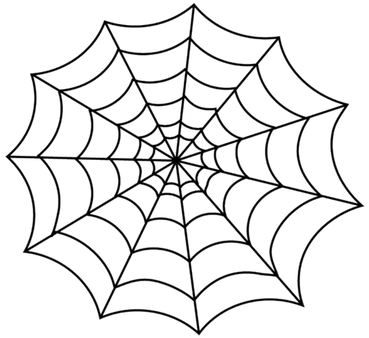

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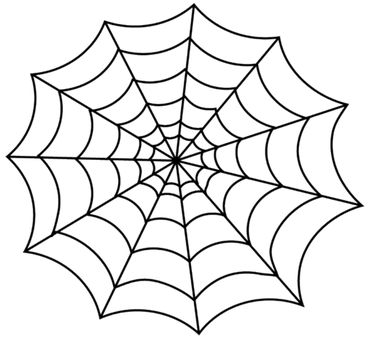

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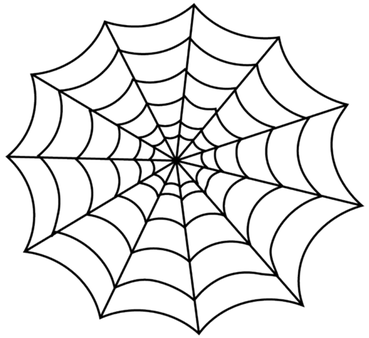

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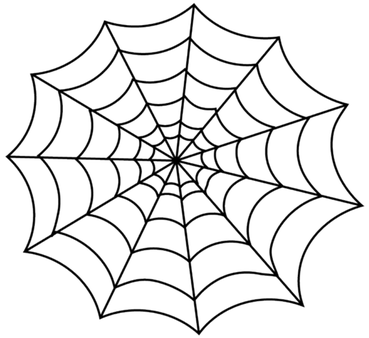
Concentrate on OR, decision trees, DNFs



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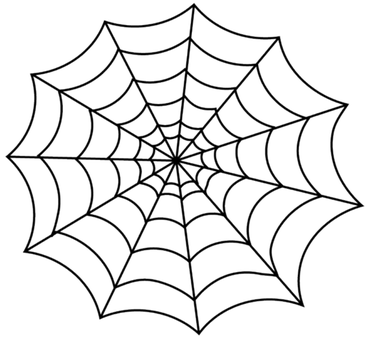

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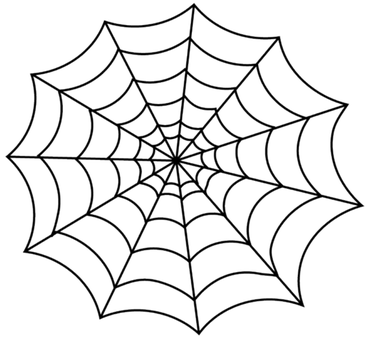

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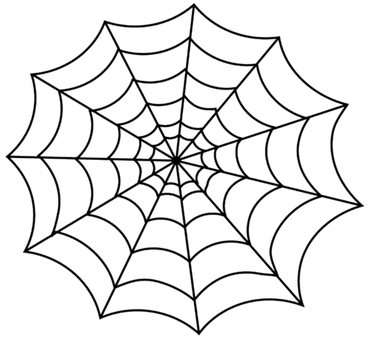

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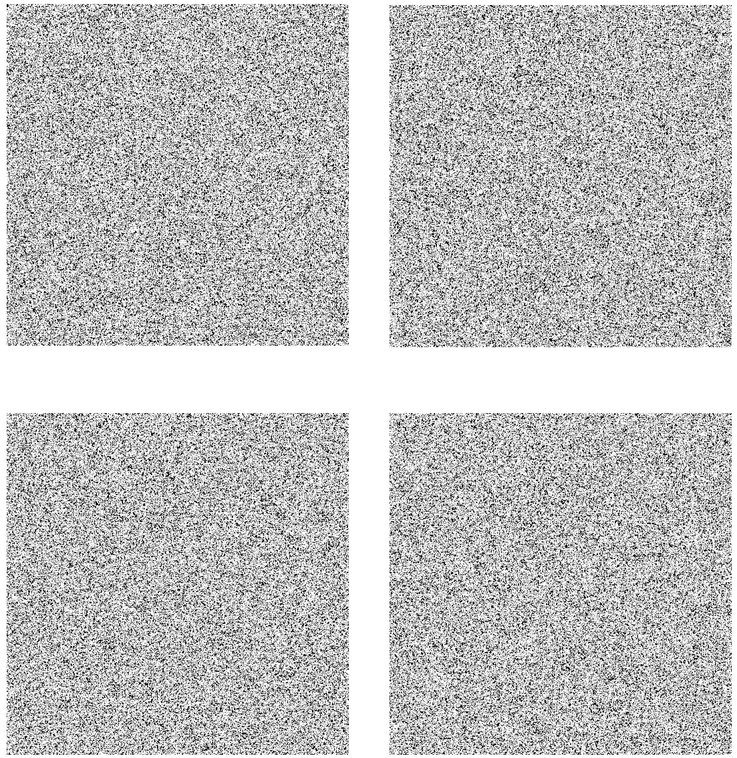

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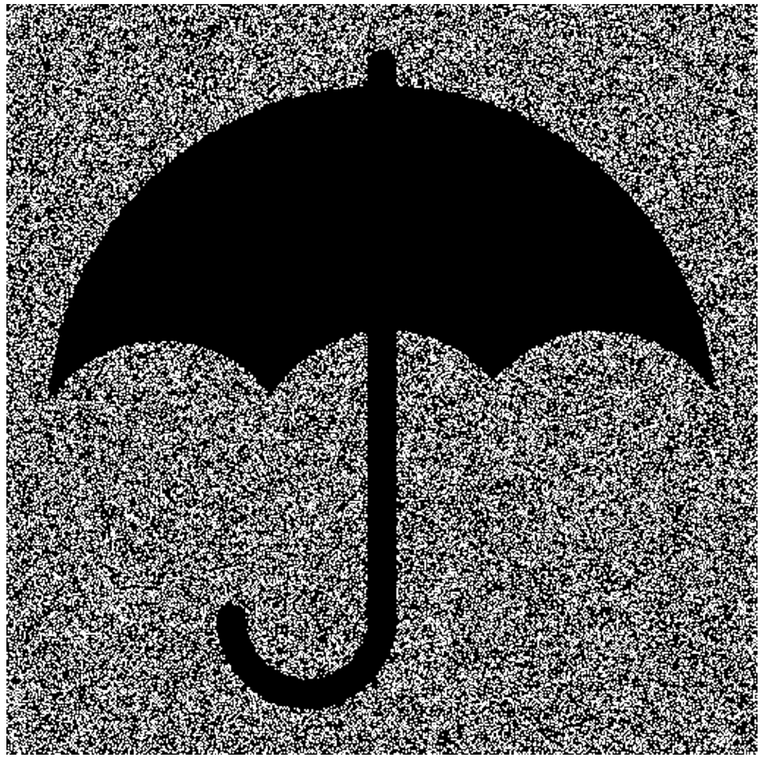

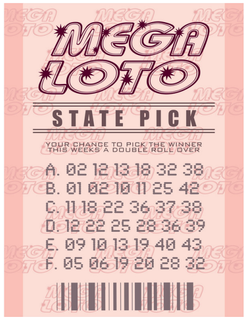

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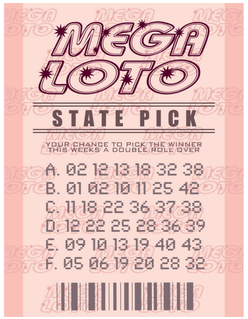
k-wise indistinguishable sources



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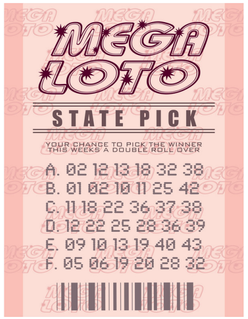

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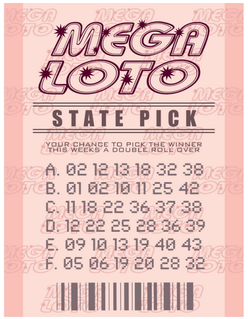
$k = \mathrm{polylog}(n)$ fools decision trees



Our Results

k-wise indistinguishable sources

Constant degree/locality

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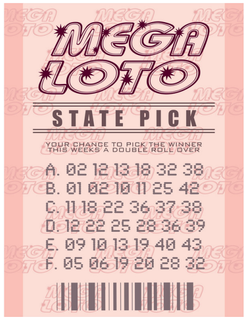

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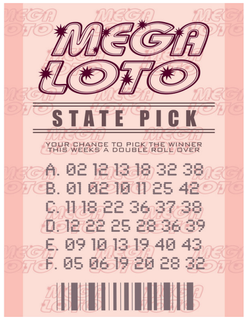

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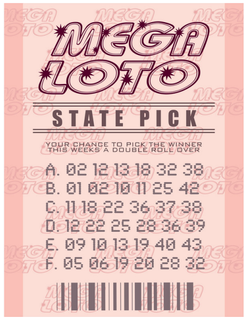

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Application to visual secret sharing


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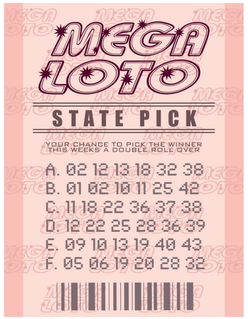

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Turns out complexity is low enough

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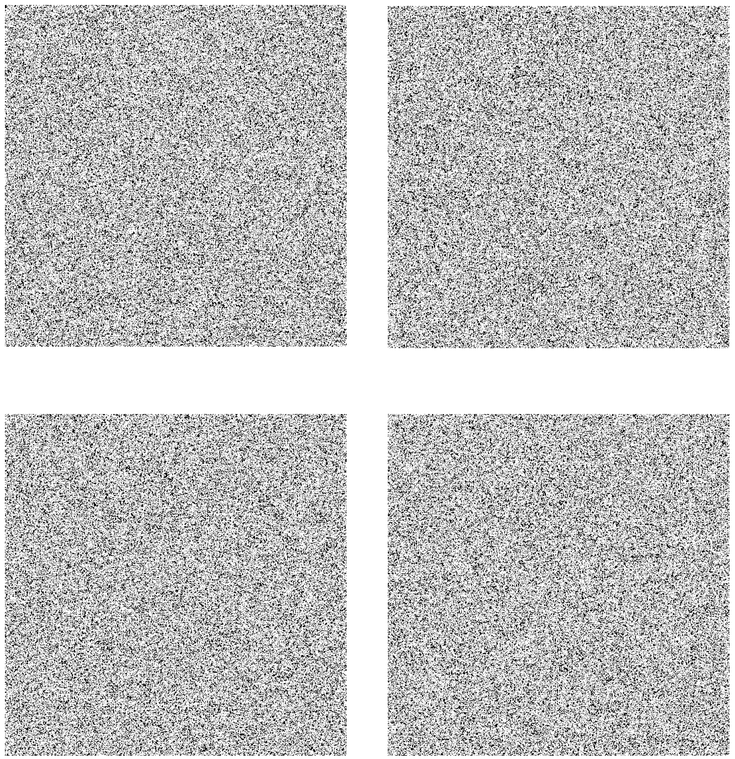
Error is 2^{-d}, so need degree $d = O(\log s)$

Application: Visual Secret Sharing



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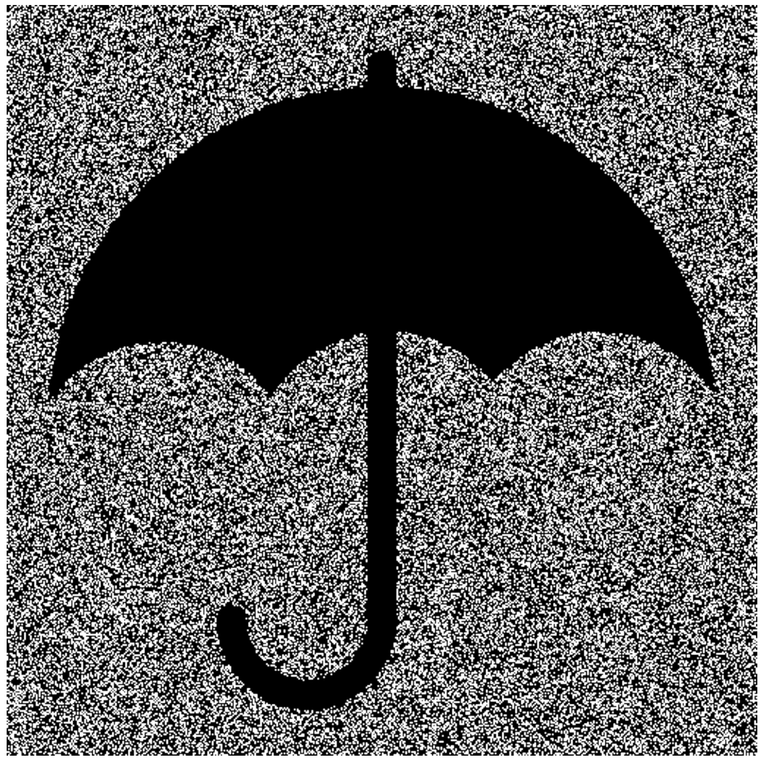

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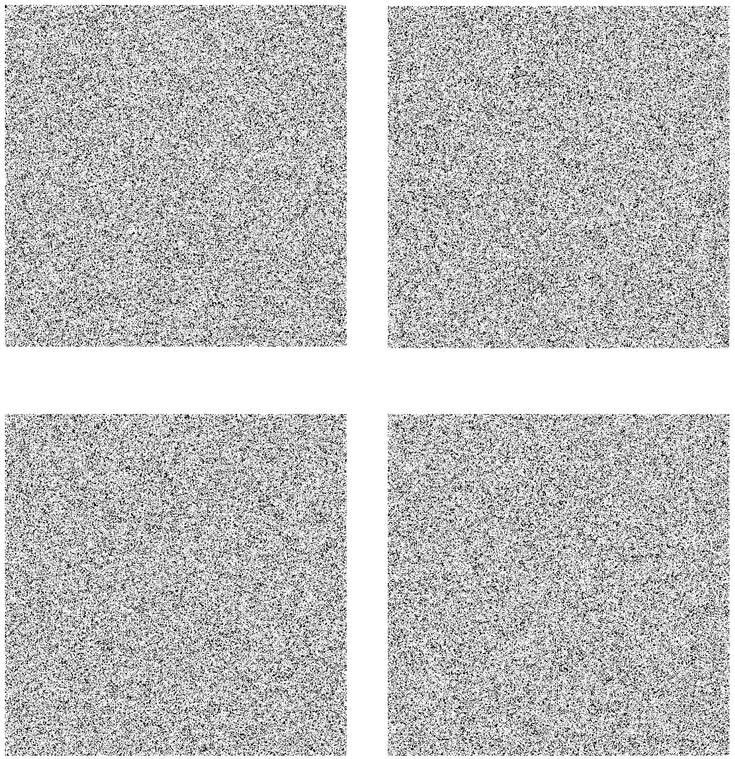

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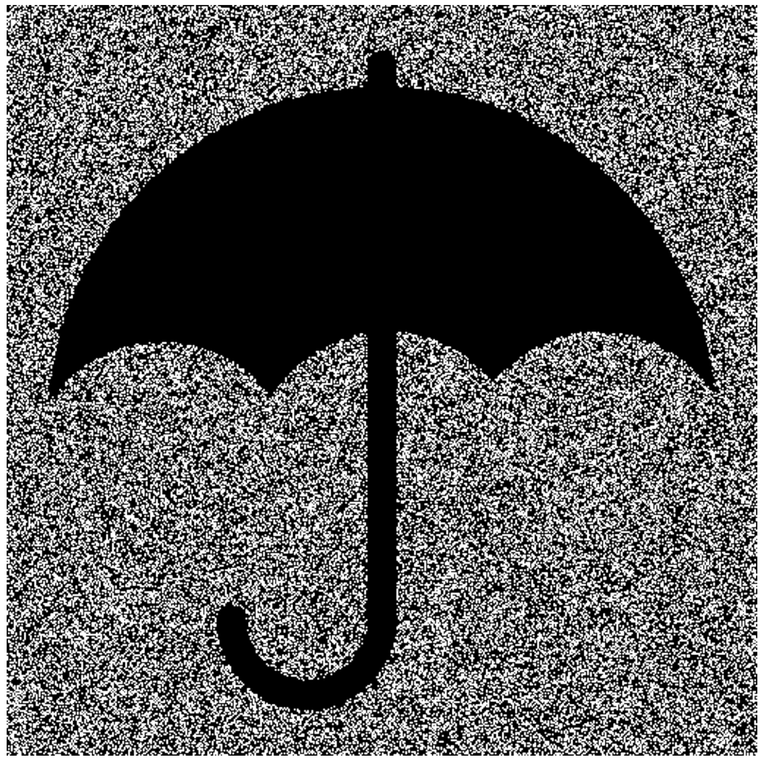

Given two distributions on biases: *X* for white, *Y* for black

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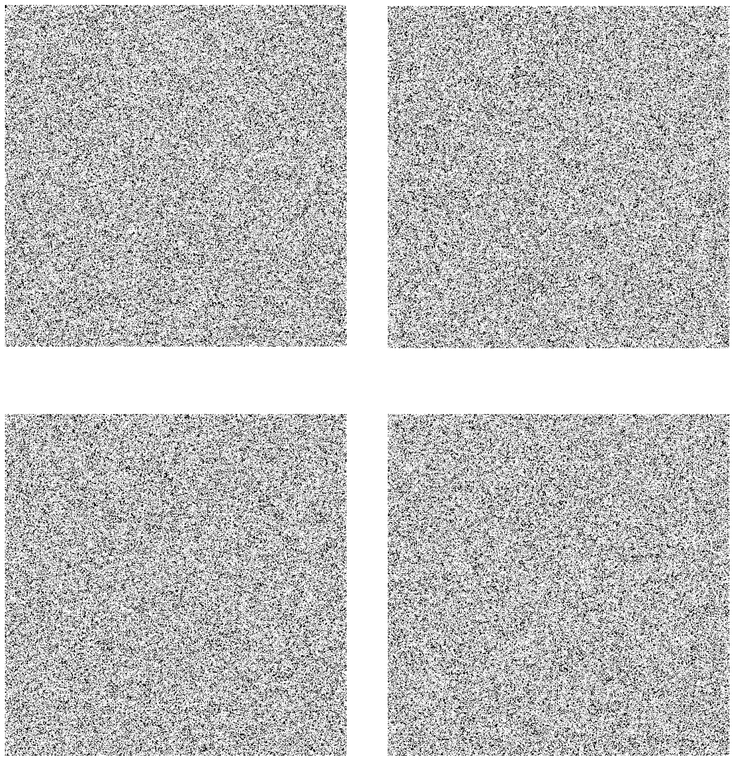

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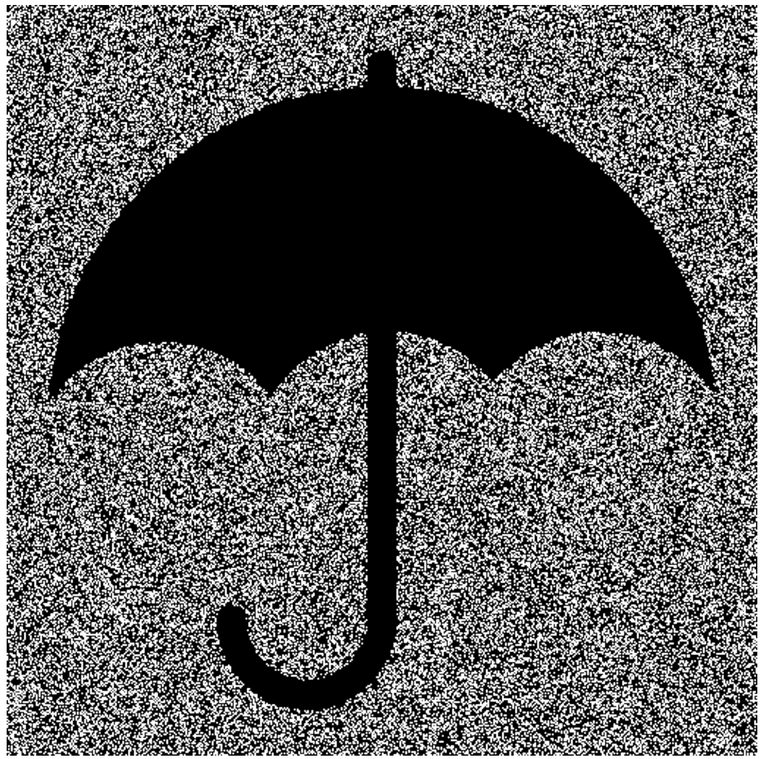

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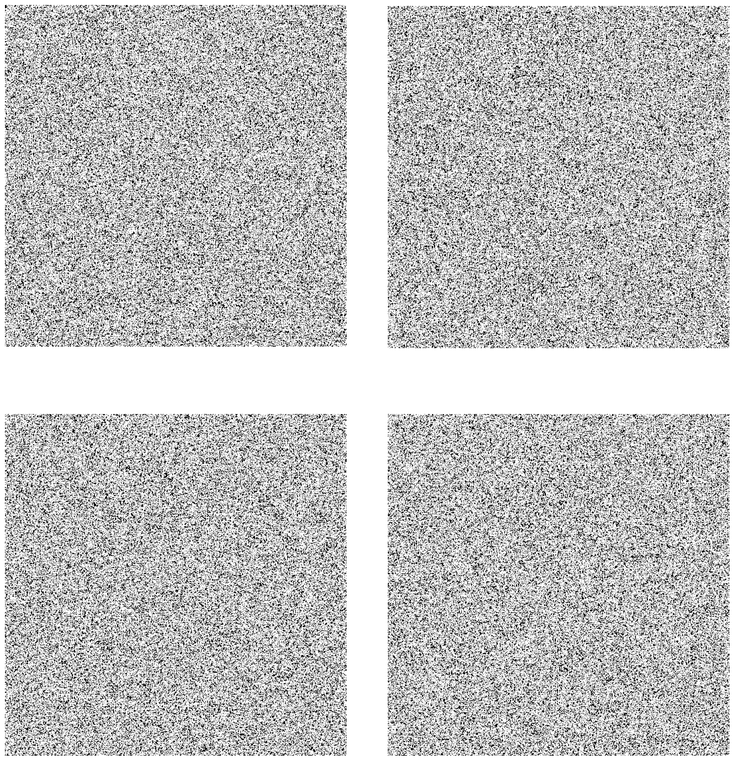

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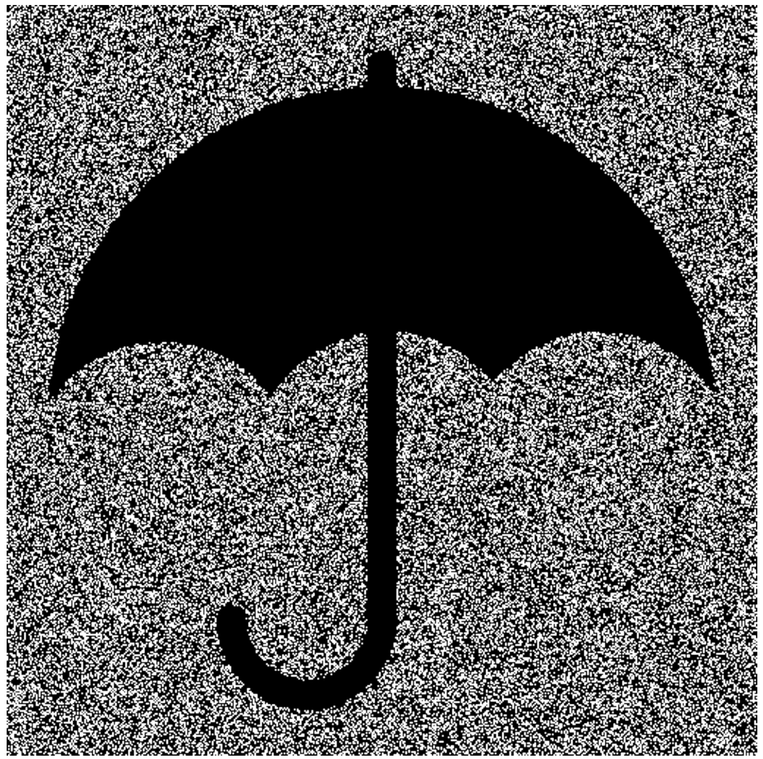

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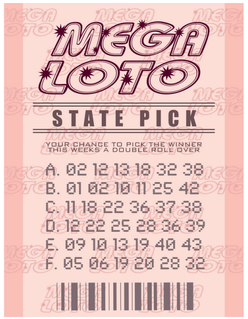

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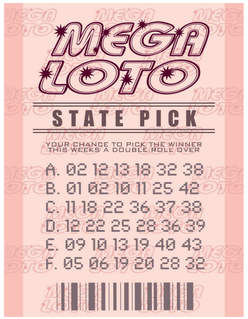
OR has advantage $\leq \epsilon \implies$ Size s decision trees have advantage $\leq s\epsilon$



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Beyond OR

Results on DNFs or AC^0? No barriers for local sources!

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Application: secret-sharing with sharing in NC^0 and reconstruction in AC^0 (current best: sharing using decision trees and reconstruction using OR)

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Web of conjectures

Given linear preprocessing $g_j(y)$, which parities of y are computable in AC^0?

Linear IPPP: not all — Our conjecture: short linear combinations — Equivalent?

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Beyond Boolean

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Connection to approximate degree breaks down

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Application: secret sharing scheme in AC^0 with "sharp threshold"