Random Graphs — Assignment 1

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Question 1. Fix a graph H, and suppose that $p = \omega(n^{-1/m(H)})$. In class, we showed that with high probability, $G(n,p)$ contains a copy of H. Show that in fact, with high probability $G(n,p)$ contains $\omega(1)$ copies of H.

Question 2. Suppose that $p = o(1/n)$. Estimate the probability that $G(n,p)$ contains a triangle by finding a function $q(n,p)$ such that

$$\Pr[G(n,p) \text{ contains a triangle}] \sim q(n,p).$$

(Note that as shown in class, $q(n,p) = o(1)$.)

Question 3. In this question we will analyze the probability that $G(n,c/n)$ contains the graph $H := \rhd\!\!-$ (a triangle with an attached edge).

(a) Let E_k be the event that $G(n,c/n)$ contains exactly k vertex-disjoint triangles, each of them isolated (not connected by an edge to the outside world), and no other triangles. Calculate $\lim_{n\to\infty} \Pr[E_k]$, and deduce a lower bound on the probability that $G(n,c/n)$ is H-free (contains no copy of H).

(b) Show that for every $\epsilon > 0$ there is k such that for large enough n, the probability that $G(n,c/n)$ contains more than k triangles is less than ϵ.

(c) Prove a matching upper bound on the probability that $G(n,c/n)$ is H-free.