Assignment 4

Random Graphs, Technion, 2017

January 12, 2018

The goal of this exercise is to generalize some results seen in class about cliques to the bipartite setting. We start with a few definitions:

- $G_2(n,p)$ is a random bipartite graph generated as follows. The vertex set consists of two sets A, B of size n each. For every $a \in A$ and $b \in B$, we add the edge $\{a, b\}$ with probability p, independently.
- Given a bipartite graph G with bipartition A, B, a biclique of size m (or, m-biclique) consists of two sets $S \subseteq A$ and $T \subseteq B$, both of size m, such that all edges between S and T belong to G.
- Given a bipartite graph G with bipartition A, B, the biclique number $\omega_2(G)$ is the maximum size of a biclique in G.
- $G_2(n,p,m)$ is a random bipartite graph generated by drawing $G \sim G_2(n,p)$, drawing sets $S \subseteq A$ and $T \subseteq B$ of size m each at random, and adding to G all edges between S and T.

All the following questions are analogs of results we have seen in class. You can also use the lecture notes, which are more detailed.

1. Let k_0 be the maximal integer such that the expected number of k_0-bicliques in $G_2(n, 1/2)$ is at least 1. Estimate k_0.
2. Determine a value k_1 such that with high probability, $G_2(n, 1/2)$ contains a k_1-biclique. Attempt to make k_1 as close to k_0 as possible.
3. Determine a value k_2 such that there exists a polynomial time algorithm that finds a k_2-biclique in $G_2(n, 1/2)$ with high probability (over the choice of the graph). Attempt to make k_2 as large as possible.
4. Show that for every $C > 0$ you can find in polynomial time an m-biclique in $G_2(n, 1/2, m)$ with high probability (over the choice of the graph), where $m = C\sqrt{n \log n}$.