

Assignment 2

Random Graphs, Technion, 2017

November 26, 2017

1. **Solving NP-hard problems on random graphs:** Fix $c < 1$. An *algorithm for the maximum clique problem* is a deterministic polynomial time algorithm which accepts a graph as an input and outputs a clique in the graph. Show that there is an algorithm for the maximum clique problem which, when run on a $G(n, c/n)$ random graph, outputs a maximum size clique with high probability. Do the same for maximum independent set, minimum vertex cover, minimum feedback vertex set, and chromatic number (“minimum coloring”).
2. **Unicyclic graphs, empirically:** Let U_n be the number of connected graphs on n vertices which have exactly n edges. The goal of this exercise is to determine U_n empirically.
 - (a) Calculate $\Pr[G(n, n) \text{ is connected}]$.
 - (b) Show how to estimate U_n given many samples of $G(n, n)$ (that is, $G(n, m)$ with $m = n$).
 - (c) Let X be the estimate of U_n obtained from M many samples. Calculate the standard deviation of X .
 - (d) Assuming that $U_n \approx n^n$, estimate how many samples are needed to determine U_n with a certainty of 99.7% (see Wikipedia).
 - (e) Calculate U_3, U_4, U_5 empirically (that is, by writing a computer program). Can you also calculate U_6 ?

Please provide the source code along with your solution.

3. **Unicyclic graphs, combinatorially:**
 - (a) Give a formula for U_n . One way is to decompose the graph into a cycle, each vertex of which is the root of a tree.
 - (b) Calculate U_3, U_4, U_5, U_6 using your formula, and compare to your findings from the preceding exercise. Can you calculate more values of U_n ?

If you used a computer program in part (b), please provide the source code along with your solution.

4. **Unicyclic graphs, using the internet:** Use an internet search to determine the asymptotic rate of growth of U_n , and compare it to your findings from the preceding exercises.