Assignment 1

Random Graphs, Technion, 2017

November 13, 2017

1. **Emptyness threshold:**
   (a) Show that if $p = o(1/n^2)$ then whp $G(n,p)$ is empty.
   (b) Show that if $p = \omega(1/n^2)$ then whp $G(n,p)$ is not empty.
   (c) What happens when $p = c/n^2$?

2. **Matching threshold:**
   (a) Show that if $p = o(1/n^{3/2})$ then whp $G(n,p)$ is a matching (disjoint union of edges).
   (b) Show that if $p = \omega(1/n^{3/2})$ then whp $G(n,p)$ is not a matching.

3. **The countable random graph:**[1]
   (a) Suppose that U is a countable universe and that ε is a binary relation on U satisfying the following properties:
      i. For every finite set $S \subseteq U$, there exists an element $s \in U$ such that $x \,\varepsilon\, s$ iff $x \in S$.
      ii. If $x_1 \,\varepsilon\, x_2 \,\varepsilon \cdots \varepsilon\, x_n$ then $x_1 \neq x_n$.

      Form a graph G on the vertex set U by connecting x, y if either $x \,\varepsilon\, y$ or $y \,\varepsilon\, x$. Show that G is saturated (for every disjoint finite $A, B \subseteq U$, there exists a vertex x connected to all vertices of A and not connected to any of the vertices in B), and so the random countable graph.
   (b) Let U be the natural numbers (starting with 0), and define $x \,\varepsilon\, y$ if the xth bit of y is 1 (the 0th bit is the least significant bit). Show that (U, ε) satisfy the two axioms listed above.

[1] This exercise was proposed by Peter J. Cameron.