1. **Emptyness threshold:**
   (a) Show that if $p = o(1/n^2)$ then whp $G(n, p)$ is empty.
   (b) Show that if $p = \omega(1/n^2)$ then whp $G(n, p)$ is not empty.
   (c) What happens when $p = c/n^2$?

2. **Matching threshold:**
   (a) Show that if $p = o(1/n^{3/2})$ then whp $G(n, p)$ is a matching (disjoint union of edges).
   (b) Show that if $p = \omega(1/n^{3/2})$ then whp $G(n, p)$ is not a matching.

3. **The countable random graph:**\(^1\)
   (a) Suppose that $U$ is a countable universe and that $\varepsilon$ is a binary relation on $U$ satisfying the following properties:
   i. For every finite set $S \subseteq U$, there exists an element $s \in U$ such that $x \varepsilon s$ iff $x \in S$.
   ii. If $x_1 \varepsilon x_2 \varepsilon \cdots \varepsilon x_n$ then $x_1 \neq x_n$.
   Form a graph $G$ on the vertex set $U$ by connecting $x, y$ if either $x \varepsilon y$ or $y \varepsilon x$. Show that $G$ is saturated (for every disjoint finite $A, B \subseteq U$, there exists a vertex $x$ connected to all vertices of $A$ and not connected to any of the vertices in $B$), and so the random countable graph.
   (b) Let $U$ be the natural numbers (starting with 0), and define $x \varepsilon y$ if the $x$th bit of $y$ is 1 (the 0th bit is the least significant bit). Show that $(U, \varepsilon)$ satisfy the two axioms listed above.

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\(^1\)This exercise was proposed by Peter J. Cameron.