# Judgment aggregation meets Property testing

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## 1 Introduction

A panel of judges has convened to adjudicate a criminal case. The accused should be charged if they have both the means and the motive. Here are the thoughts of the various judges:

| Means | Motive | Guilty |
|---|---|---|
| Yes | Yes | Yes |
| Yes | No | No |
| No | Yes | No |

A majority of judges think the accused had the means and the motive, yet a majority of them think they are not guilty! This shows that Majority is not a valid judgment aggregation function in this setting.

A *judgment aggregation function* for $n$ judges is a function $f\colon \{0,1\}^n \to \{0,1\}$ (where 0 stands for No and 1 stands for Yes). A judgment aggregation function is *valid* if for all $x, y \in \{0,1\}^n$,

$$f(x_1 \wedge y_1, \ldots, x_n \wedge y_n) = f(x_1, \ldots, x_n) \wedge f(y_1, \ldots, y_n).$$

Here $x$ represents the thoughts of the judges on the means, and $y$ their thoughts on the motive; thus $x \wedge y$ is their verdict. The function $f$ is used to aggregate each of these dimensions into a single truth value. A valid function guarantees that the final judgment is sound: the accused is convicted iff they have both the means and the motive.

The example above shows that Majority is not a valid aggregation function. One aggregation function which is valid is the projection $f(x_1, \ldots, x_n) = x_i$, which simply outputs the opinion of the $i$th judge. In this context, this function is known as a *dictator*. More generally, for any subset $I$ of judges, the function $f(x_1, \ldots, x_n) = \bigwedge_{i \in I} x_i$ is also valid. This function, known as an *oligarchy*, outputs a Yes on one of the issue only if all judges in $I$ unanimously agree on the Yes opinion.

Apart from the constant functions, oligarchies (which generalize dictatorships) are the only valid aggregation functions for this scenario. This state of affairs is not optimal — we would prefer using Majority-like functions for aggregating judgments. One can therefore ask whether there is a judgment aggregation functions which works in *most* cases. This question, proposed by Nehama (who gave a partial answer), is the subject of this work.

## 2 Two more examples

Let us now give two more examples of this phenomenon, one from social choice theory, and the other from property testing.

### 2.1 Arrow's theorem

Consider an elections between three candidates, A, B, and C. Each voter has to rank the three candidates, and the outcome should also be a ranking of the three candidates. We want our aggregation function to satisfy a condition known as *independence of irrelevant alternatives*: the relative ranking of A and B in the final ranking should only depend on the voters' relative ranking of A and B, and similarly for all other pairs of candidates. In other words, we can think of each vote as a list of three truth values:

| $A > B$? | $B > C$? | $C > A$? |
|---|---|---|
| T | F | T |
| F | F | T |

# 3 Wider context

Arrow's theorem.

AND as judgement aggregation.

The example of XOR.

## 3.1 The universal algebra view

## 3.2 Robust versions

Kalai's version of Arrow's theorem.

Corresponding result for XOR.

What about AND?

## 3.3 The property testing view

# 4 Techniques

## 4.1 Local correction

## 4.2 Fourier analysis

# 5 Our results

First step: reduction to "noise operator".

Second step: the function is a junta.

Third step: generalized eigenvalue problem (?).